Real Numbers

Real Numbers are combination of five different types of numbers as described in detail below.

- **Natural Numbers**, commonly referred to as **counting numbers** i.e. 1, 2, 3, 4, 5, 6, 7, ...etc.
- **Whole Numbers** are combination of natural numbers and zero (0) i.e. 0, 1, 2, 3, 4, 5, 6, 7, ...etc.
- **Integers** are combination of Natural and Whole Numbers and their negatives.
  For e.g. …-7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7... Continue infinitely in both directions.
- **Rational Numbers** are made of ratios of integers and referred to as fractions. Rational numbers take on the general form of a/b where a and b can be any integer except $b \neq 0$. Thus, the Rational numbers include all Integers, Whole numbers and Natural numbers.
- **Irrational Numbers** are made of special, unique numbers that cannot be represented as a ratio of Integers. Examples of Irrational numbers include π(3.14159265358979...) and the square root of 2 (1.4142135623730950...).

Algorithm -An algorithm is a series of well defined steps which gives a procedure for solving a type of problem.

Lemma - A lemma is a proven statement used for proving another statement.

Euclid's Division lemma:

For any two given positive integers a and b there exist unique integers q and r such that $a=bq + r$, where $0\leq r<b$.

Here a, b, q and r are respectively called as dividend, divisor, quotient and remainder. Euclid's Division Lemma can be used to find H.C.F of two positive integers.

Euclid's division Algorithm: It is a technique to compute Highest Common Factor(H.C.F) of two given positive integers, consider c and d are two positive integers, with $c > d$. We use following the steps to find H.C.F of c and d:

Step I: Apply Euclid's division lemma, to c and d, so we find whole numbers, q and r such that $c = dq + r$, $0\leq r<d$

Step II: If $r=0$, d is the H.C.F of c and d. If $r\neq 0$ apply division lemma to d and r

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Step III: Continue the process till the remainder is zero. The divisor at this stage will be the required H.C.F

The Fundamental theorem of Arithmetic:

Every composite number can be expressed (factorised) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

Ex. 28 = 2 x 2 x 7 ; 27 = 3 x 3 x 3

Theorem : Sum or difference of a rational and irrational number is irrational.

Theorem : The product and quotient of a non-zero rational and irrational number is irrational.

Theorem : If p is a prime and *p* divides a^2, then *p* divides *"a"* where *a* is a positive integer.

Theorem : If p is a prime number then $\sqrt{p}$ is an irrational number.

Theorem: Let *x* be a rational number whose decimal expansion terminates. Then x can be expressed in the form of $\frac{p}{q}$ where *p and q* are co-prime and the prime factorisation of *q* is the form of $2^n.5^m$ where n, m are non negative integers.

Example: $0.7 = \frac{7}{10} = \frac{7}{2^1 \times 5^1} = \frac{7}{2 \text{ x } 5}$

Theorem: Let $x = \frac{p}{q}$ be a rational number such that the prime factorisation of *q* is not of the form of $2^n.5^m$, where *n, m* are non negative integers. Then *x* has a decimal expansion which is non terminating repeating (recurring).

Example: $\frac{7}{6} = \frac{7}{2 \times 3} = 1.1666 \ldots\ldots$

Theorem: For any two positive integers p and q, HCF (*p, q*) x LCM (*p, q*) = *p x q*

Example: 4 & 6; HCF (4, 6) = 2, LCM (4, 6) = 12; HCF x LCM = 2 x 12 =24

$\therefore p \text{ x } q = 24$

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Types of Decimals: