

## Polynomials

**Polynomial:** An algebraic expression of the form  $p(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$  where  $a_n \neq 0$  is called a polynomial in variable  $x$  of degree  $n$ . Where  $a_0, a_1, \dots, a_n$  are real numbers and each power of  $x$  is a non-negative integer.

Example:-  $2x^2 - 5x + 1$  is a polynomial of degree 2.

Note:  $\sqrt{x} + 3$  is not a polynomial.

**Degree of a Polynomial:** The highest power of  $x$  in a polynomial  $p(x)$  is called the degree of polynomial.

### Different types of Polynomial:

- ❖ **Constant Polynomial:** A polynomial of degree zero is called a constant polynomial and it is of the form of  $p(x) = k$
- ❖ **Linear Polynomial:** A polynomial of degree 1 is called a linear polynomial and it is of the form  $p(x) = ax + b$ , where  $a, b$  are real numbers and  $a_n \neq 0$ .  
**For example:  $3x - 3, 5x$ , etc.**
- ❖ **Quadratic Polynomial:** A polynomial of degree 2 is called a Quadratic Polynomial and it is of the form of  $p(x) = ax^2 + bx + c$ , where  $a, b, c$  are real numbers and  $a_n \neq 0$ .  
**For example:  $2x^2 + x - 1$**
- ❖ **Cubic Polynomial:** A polynomial of degree 3 is called a Cubic Polynomial and it is of the form of  $p(x) = ax^3 + bx^2 + cx + d$ , where  $a, b, c, d$  are real numbers and  $a_n \neq 0$ .  
**For example:  $x^3 - 1$**
- ❖ **Bi-quadratic Polynomial:** A polynomial of degree 4 is called a Bi-quadratic Polynomial and it is of the form of  $p(x) = ax^4 + bx^3 + cx^2 + dx + e$ , where  $a, b, c, d$  and  $e$  are real numbers and  $a_n \neq 0$ .

**Value of a Polynomial:** The value of a polynomial  $p(x)$  at  $x = \alpha$  in the given polynomial and is denoted by  $p(\alpha)$ .

**Graph of Polynomial:**

- ❖ Graph of a linear polynomial  $p(x) = ax + b$  is a straight line
- ❖ Graph of a quadratic polynomial  $p(x) = ax^2 + bx + c$  is a parabola which open upwards like  $\cup$  if  $a > 0$
- ❖ Graph of a quadratic polynomial  $p(x) = ax^2 + bx + c$  is a parabola which open downwards like  $\cap$  if  $a < 0$
- ❖ In general, a polynomial  $p(x)$  of degree  $n$  crosses the  $x$ - axis, at most  $n$  points.

**Zeroes of a polynomial:**

- ❖  $\alpha$  is said to be zero of a polynomial  $p(x)$  if  $p(\alpha) = 0$ . The zeros of polynomial  $p(x)$  are actually the  $X$ - coordinates of the points where the graph of  $y = p(x)$  intersects the  $X$ -axis.
- ❖ A polynomial of degree “ $n$ ” can have at most  $n$  zeros. For example: A linear polynomial has only one zero, a quadratic polynomial has two zeroes and a Cubic polynomial has three zeroes.

**Division Algorithm for Polynomials:**

- ❖ If  $p(x)$  and  $g(x)$  are any two polynomials with  $g(x) \neq 0$  then we can find polynomials  $q(x)$  and  $r(x)$  such that
$$P(x) = g(x) \times q(x) + r(x)$$
i.e. Dividend = Divisor  $\times$  Quotient + Remainder  
where,  $r(x) = 0$  or degree of  $r(x) <$  degree of  $g(x)$ . This result is known as the division algorithm for polynomials.