PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Algebraic Expression: A combination of constants and variables, connected by four fundamental arithmetic operations of +, -, x and ÷ is called an algebraic expression.

Equation: An algebraic expression with equal to sign (=) is called an equation. Without an equal to sign, it is considered as an expression only.

Linear Equation: If the greatest exponent of the variable(s) in an equation is one, then the equation is said to be a linear equation.

If number of variable used in linear equation is one, then equation is said to be linear equation in two variables.

Linear Equation in Two Variables:
An equation which can be put in the form $ax+by+c=0$, where a, b and c are real numbers, and $a \neq 0, b \neq 0$ is called a linear equation in two variables x and y.

General Form of a Pair of Linear Equation in Two Variables:

❖ A pair of linear equations in two variables x and y can be represented as follows:

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$,

where a_1, a_2, b_1, b_2, c_1, c_2 are real numbers such that $a_1^2 + b_1^2 \neq 0$, $a_2^2 + b_2^2 \neq 0$

❖ **System of simultaneous linear equations:**

Consistent system: A system of simultaneous linear equations is said to be consistent if it has at least one solution.

Inconsistent system: A system of simultaneous linear equations is said to be inconsistent if it has no solution.

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- **A pair of linear equations in two variables can be solved by the:**

  (i) Graphical Method.

  (ii) Algebraic Method.

  Algebraic methods are of three types:

  (a) Substitution Method.

  (b) Elimination Method.

  (c) Cross-multiplication Method.

Graphical Method:

(i) Intersecting, if $\frac{a1}{a2} \neq \frac{b1}{b2}$

here, the equations have a **unique solution,** and pair of equations is said to be **consistent.**

(ii) Parallel, if $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$

here, the equations have **no solution**, and pair of equations is said to be **inconsistent.**

(iii) Coincident, if $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$

here, the equations have **infinitely many solutions**, and pair of equations is said to be **consistent.**

Algebraic Method:

a) Substitution Method:

Let us consider any two equations

$\frac{2x}{a} + \frac{y}{b} = 2..........(A)$

$\frac{x}{a} - \frac{y}{b} = 4..........(B)$

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Get the value of x in terms of y from equation (A), Substitute the value of x in equation (B) to get value of y.

b) Elimination Method:

Consider two equations

$$\frac{2x}{a}+\frac{y}{b}=2..........(A)$$

$$\frac{x}{a}-\frac{y}{b}=4..........(B)$$

Step1: Comparing the coefficients of x and y.

$$2bx+ay=2ab.......(i)$$

$$bx-ay=4ab.......(ii)$$

Step2: Simplify the equation either by adding or subtracting.

Step 3: After simplifying you will find one value either ‘x’ or ‘y’.

Step4: Substitute value of ‘x’ or ‘y’ in any of one equation to get the value of other variable.

c) Cross Multiplication Method:

Consider two equations:

$$\frac{2x}{a}+\frac{y}{b}=2..........(A)$$

$$\frac{x}{a}-\frac{y}{b}=4..........(B)$$

Step1:

- ❖ Below ‘x’ write the coefficients of ‘y’ and the constant terms.
- ❖ Below ‘y’ write the coefficients of ‘x’ and the constant terms.
- ❖ Below 1, write the coefficients of ‘x’ and ‘y’.

Step2:

- Simplify the equations.

Step3:

- After simplifying equate the values of 'x' and 'y' with constant terms, we get value of 'x' and 'y'.

Equation Reducible to a Pair of Linear Equations In Two Variables

There are several situations which can be mathematically represented by two equations that are not linear to start with. But we alter them so as to reduce to pair of linear equations.

We will be easily able to understand this concept of equation reducible to a pair of linear equation in two variables..

We shall discuss the solution of such pairs of equations which are not linear but can be reduced to linear form by making some suitable substitutions. We now explain this process through some examples.

Example: Solve the pair of equations:

$$\frac{2}{x} + \frac{3}{y} = 13$$

$$\frac{5}{x} - \frac{4}{y} = -2$$

Solution: Let us write the given pair of equations as

$$2\left(\frac{1}{x}\right) + 3\left(\frac{1}{y}\right) = 13 \qquad (1)$$

$$5\left(\frac{1}{x}\right) - 4\left(\frac{1}{y}\right) = -2 \qquad (2)$$

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These equations are not in the form ax + by + c = 0. However, if we

Substitute $\frac{1}{x}$ = p and $\frac{1}{y}$ = q in Equations (1) and (2) we get,

2p + 3q = 13 -------------------------------------- (3)

5p - 4q = -2 -- (4)

So, we have expressed the equations as a pair of linear equations. Now, you can use any method to solve these equations, and get p = 2 and q = 3.

We know that p = $\frac{1}{x}$ and q = $\frac{1}{y}$

Substitute the values of p and q to get $\frac{1}{x}$ = 2 i.e. x = $\frac{1}{2}$ and $\frac{1}{y}$ = 3 i.e. y = $\frac{1}{3}$

Verification: By substituting x = $\frac{1}{2}$ and y = $\frac{1}{3}$ in the given equations, we find that both the equations are satisfied.