

QUADRATIC EQUATIONS

(Quadratic equation is an equation in which the highest power of an unknown variable is 2)

1. Quadratic Equation:

Any equation of the form $p(x) = 0$, where $p(x)$ is a polynomial of degree/power 2, is a quadratic equation.

Example:

$$2x^2 + x - 300 = 0,$$

$$1 - x^2 + 300 = 0,$$

$$4x + 2 - 3x^2 = 0, \text{ etc.}$$

2. Standard Form of Quadratic Equation:

When we write the terms of $p(x)$ in descending order of their degrees, then we get the standard form of the equation i.e.

$$ax^2 + bx + c = 0,$$

where a, b, c are real numbers, $a \neq 0$.

3. Methods to find roots/solutions of a quadratic equation:

- a. Factorization method
- b. Completing the square method
- c. Discriminant method

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4. Solution of a Quadratic Equation by Factorization:

- a. A real number α is said to be a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$ we can say that $x = \alpha$ is a solution of the quadratic equation

Note:

Zeroes of the quadratic polynomial $ax^2 + bx + c$

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Roots of the quadratic equation $ax^2 + bx + c = 0$

- b. If we factorize $ax^2 + bx + c$, $a \neq 0$, into a product of **two linear factors**, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to **zero**.

Example:

The roots of $6x^2 - x - 2 = 0$ are the values of x for which

$$(3x - 2)(2x + 1) = 0$$

$$(3x - 2) = 0 \quad \text{or} \quad (2x + 1) = 0$$

$$x = \frac{2}{3}$$

or

$$x = \frac{-1}{2}$$

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5. Solution of a Quadratic Equation by Completing The Square:

PROCESS OF COMPLETING THE SQUARE

e.g $x^2 + 2x - 1 = 0$
 $x^2 + 2x = 1$

$$x^2 + 2x + (\square) = 1 + (\square)$$



$$\frac{1}{2} \times 2 = 1 = (1)^2 = 1$$

$$x^2 + 2x + (1) = 1 + (1)$$

$$\because a^2 + 2ab + b^2 = (a + b)^2$$

$$(x + 1)^2 = 2$$

$$x + 1 = \pm \sqrt{2}$$

$$x = -1 \pm \sqrt{2}$$

$$x = -1 + \sqrt{2} \text{ or } x = -1 - \sqrt{2}$$

1) Keep all terms containing x on one side. Move the constant to the other.

2) Need to create a perfect square on the L.H.S. Balance the equation.

3) Use the formula :

$$\text{Last term} = \left(\frac{1}{2} \times \text{coefficient of the } x\text{-term}\right)^2$$

4) Add this value to both the sides.

5) Simplify & write the perfect square on the L.H.S based on identity.

6) Take the square root throughout & consider both plus and minus signs.

7) Solve for x .

Note:

For equations with coefficient of x^2 other than 1, divide the whole equation by the same number on both the sides to get 1 as the coefficient of x^2 and then start the process of completing the square.

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6. Discriminant:

A discriminant of a quadratic equation determines whether the quadratic equation $ax^2 + bx + c = 0$ has real roots or not.

Note: Check point no. 7: Nature of the roots

$$\text{DISCRIMINANT} = b^2 - 4ac$$

7. Quadratic Formula:

The roots of a quadratic equation $ax^2 + bx + c = 0$ are given by

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where, **DISCRIMINANT** ≥ 0 (Discriminant = $b^2 - 4ac$)

8. Nature of the roots:

A quadratic equation $ax^2 + bx + c = 0$ has

- a. Two distinct real roots, if $b^2 - 4ac > 0$
- b. Two equal roots, if $b^2 - 4ac = 0$
- c. No real roots, if $b^2 - 4ac < 0$