CIRCLES

Circle: A circle is a collection of all points in a plane which are at a constant distance from a fixed point.

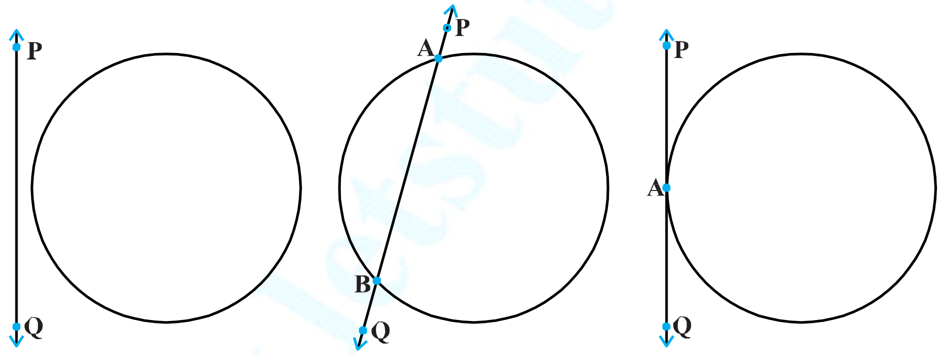
Some parts of a circle:

Chord: A line segment joining any two end points on the circle is called a chord. Diameter is the longest chord.

Secant: A line which intersects the circle in two distinct points is called secant of the circle.

Tangent: A line which touches the circle at only one point is called tangent to the circle.

Three different situations that can arise when a circle and a line are given in a plane, consider a circle and a line PQ. There can be three possibilities as given:



(i) (ii) (iii)

(i) The line PQ and the circle have no common point. Here, PQ is called a **non-intersecting** line with respect to the circle.

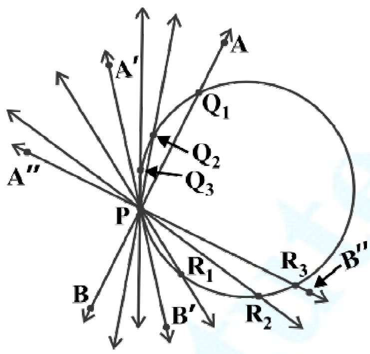
(ii) There are two common points A and B that the line PQ and the circle have. Here, we call the line PQ a **secant** of the circle.

(iii) There is only one point A which is common to the line PQ and the circle. Here, the line PQ is called a **tangent** to the circle.

Tangent to the circle:

❖ A tangent to a circle is a line that intersects the circle at only one point.

❖ The word 'tangent' comes from the Latin word 'tangere', which means to touch and was introduced by the Danish Mathematician Thomas Fineke in 1583.

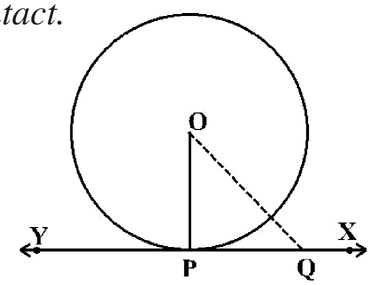


❖ In the above figure, there is only one tangent at a point of the circle.
A' B' is a tangent to the circle.

❖ The tangent to a circle is a special case of the secant, when the two end points of its corresponding chord coincide.

Important Theorems:

Theorem 1: *The tangent at any point of a circle is perpendicular to the radius through the point of contact.*



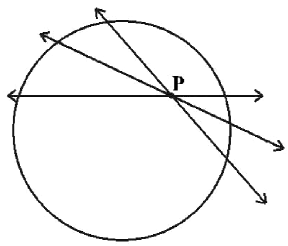
Remarks:

1. By theorem mentioned above, we can also conclude that at any point on a circle there can be one and only one tangent.

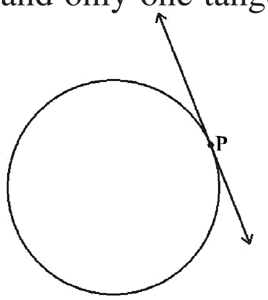
2. The line containing the radius through the point of contact is also sometimes called the '**normal**' to the circle at the point.

Number of Tangents from a Point on a Circle:

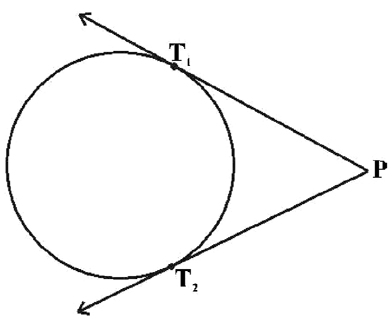
- **Case 1:** There is no tangent to a circle passing through a point lying inside the circle.



- **Case 2:** There is one and only one tangent to a circle passing through a point lying on the circle

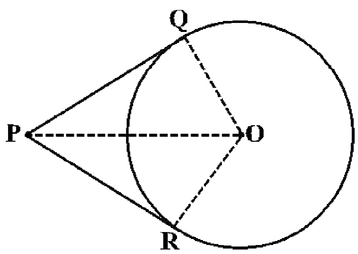


- **Case 3:** There are exactly two tangents to a circle through a point lying outside the circle



Note: The length of the segment of the tangent from the external point P and the point of contact with the circle is called the **length of the tangent** from the point P to the circle.

Theorem 2: *The lengths of tangents drawn from an external point to a circle are equal.*



From the above diagram, i.e. in right triangles OQP and ORP,
OQ = OR (Radius of the same circle with centre O)
OP = OP (Common side for both the triangles)
Also, $\angle OQP = \angle ORP$
$\Rightarrow$ By RHS congruence condition,

$$\Delta OQP \cong \Delta ORP$$

Which gives us PQ = PR (Corresponding side of congruent triangles)

Remarks:

1. The theorem can also be proved by using the Pythagoras Theorem as follows: $PQ^2 = OP^2 - OQ^2 = OP^2 - OR^2 = PR^2$ (As OQ = OR) which gives PQ = PR.

2. Note that $\angle OPQ = \angle OPR$. Therefore, OP is the angle bisector of $\angle QPR$, i.e. the centre lies on the bisector of the angle between the two tangents.